Modeling Vocal Fold Tissue: Two- and Three-Network Ogden Models

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The contents of this memo provide: (1) a brief overview of two- and three-network viscoelastic Ogden models for tissue modeling; (2) numerical implementation of the models in MATLAB using a general differential equation solver; and (3) some insights into using the model (e.g., parameter sensitivities, singularities, solutions). The goal was to set the stage for the potential translation of the model into FORTRAN so that it could be eventually imbedded into the Titze SPEAK model, a voice and speech simulator. All scripts and updates to this memo can be downloaded at http://www.ncvs.org/research_techbriefs.html .

Keywords: viscoelastic, muscle models, tissue models, larynx, vocal fold, optimization, muscle model, passive characteristics, Young's modulus, stress-strain, material properties, mechanical properties.

List of Symbols:

Symbol	Description
μ_A	*parameter*, initial shear modulus of the equilibrium network (A),
α	*parameter*, dimensionless constant describing the nonlinearity of the elastic response
$\dot{\lambda}_B^e$	*function*, Network (B) elastic stretch rate
m	*parameter*, dependence of inelastic deformation on network (B)
c	*parameter*, dependents of rate of inelastic deformation on current magnitude of inelastic deformation
Z	*parameter*, viscousity scaling constant
δ	*parameter*, small positive number (<0.001) to avoid singularities
λ_u	*independent variable*, applied stretch (fractional elongation)
$\dot{\lambda}_u$	*calculated variable*, elastic stretch rate
σ	*dependent variable*, stress (force per unit area)

Introduction

The passive axial properties of laryngeal tissues are key components to global shaping and displacement of the larynx (Hunter, Titze, & Alipour, 2004), as well as to pitch control (Titze, 1994). To better understand how these properties relate to shaping, displacement and pitch control, large-scale biomechanical models have been created (Hunter et al., 2004; Titze & Hunter, 2007) which rely on tissue models of the axial properties. The axial properties of laryngeal tissue are both highly nonlinear (Hunter, Alipour, & Titze, 2007; Hunter et al., 2007; Hunter & Titze, 2007) as well as viscous (rate dependent and hysteretic to cyclic loading) (Chan, Fu, Young, & Tirunagari, 2007). The nature of the material has two primary time constants of creep, with a shorter term creep captured both as a short effect (Alipour-Haghighi & Titze, 1985; Chan et al., 2007; Hunter & Titze, 2007; Zhang, Siegmund, Chan, & Fu, 2009) and a longer term stress relaxation (Alipour & Titze, 1999; Alipour-Haghighi & Titze, 1985; Chan, Siegmund, & Zhang, 2009). This report focuses primarily on an Ogden-type model with two networks(Chan et al., 2007, 2009; Zhang et al., 2009) (shorter term creep) and three networks (long-term creep) (Chan et al., 2009). The purpose was three fold:

1. To implement these models in MATLAB without using MATLAB's internal differential numerical equation solvers—allowing for more portability to other programming languages—and provide the resulting scripts.
2. To implement the models in FORTRAN for general portability, as well as with the specific purpose to implement them eventually in a larger simulator of vocal fold posturing and speech (Hunter et al., 2004; Titze & Hunter, 2007).
3. To investigate the Ogden-type models' sensitivity to experimental error and to previous models of axial properties (Hunter & Titze, 2007) using an automatic parameter optimization strategy to search for parameters matching actual data.

Materials and Methods

This section is divided into several parts: 1) a quick review of the Ogden tissue models; 2) explanation of the implementation of the models in MATLAB and FORTRAN, and 3) description of the optimization of the models' parameters, as well as the sensitivities of these parameters to experimental error.

1. REVIEW OF THE OGDEN TISSUE MODELS

The Ogden model, described in more detail elsewhere (Chan et al., 2007, 2009), is a one-dimensional rheological model based on principles of continuum. The most basic Ogden model has a similar look as the classic Kelvin model with two parallel networks. Network one consists of a single compliant term (spring) and a parallel Maxwell element network (consisting of a compliant component in series with a resistive element). This will be called the Ogden *two-network* model with the compliant element having hyperelastic characteristics (Network A) and the Maxwell element being viscoplastic (Network B). Adding more complexity, an additional Maxwell element (Network C), resulting in the *three-network* model (Figure 12.1), was introduced into the model. The two- and three-network models have been discussed previously (Chan et al., 2007, 2009). Adding an infinite number of Maxwell elements is a common way to model viscoelastic creep and has been used to model the relaxation function in vocal fold materials (Alipour-Haghighi & Titze, 1985). The basic equations and parameters of the two-network Ogden model will be reviewed briefly. Further descriptions of the models can be found in the referenced papers.

The two-network model consists of a time-independent equilibrium Network A in parallel with a time-dependent Network B. The total nominal stress in network A is given as Equation 12.1,

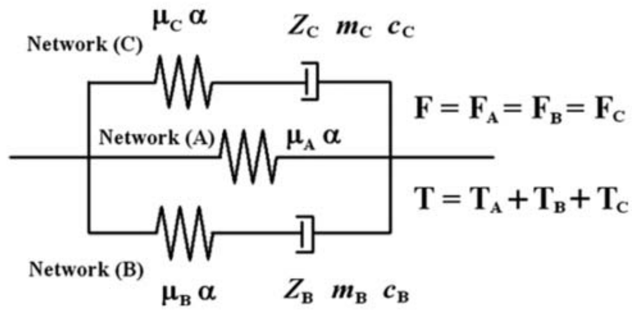


Figure 12.1. A one-dimensional rheological schematic of the three-network Ogden model. Removing Network C would result in the two-network Ogden model. Figure after Chan *et al* 2009[11].

$$\sigma = \sigma_A = \frac{2\mu_A}{\alpha}\left(\lambda_u^{\alpha-1} - \lambda_u^{-\frac{\alpha}{2}-1}\right), \qquad (12.1)$$

where μ_A is the initial shear modulus of the equilibrium Network A, α is the corresponding dimensionless constant describing the nonlinearity of the elastic response and λ_u is the applied stretch (note that *stretch* is not *strain*). For Network A, the elastic stretch rate is $\dot{\lambda}_u$.

For Network B, the elastic stretch rate $\dot{\lambda}_B^e$ is given as:

$$\dot{\lambda}_B^e = \dot{\lambda}_u \frac{\lambda_B^e}{\lambda_u} - \frac{2}{3}\lambda_B^e \cdot Z \cdot sgn(\lambda_B^e - 1) \cdot \left\{\sqrt{\frac{1}{3}\left[\left(\frac{\lambda_u}{\lambda_B^e}\right)^2 + \frac{2\lambda_B^e}{\lambda_u}\right]} - 1 + \delta\right\}^c \cdot \left|\frac{2\mu_B}{\alpha}\left[(\lambda_B^e)^\alpha - (\lambda_B^e)^{-\alpha/2}\right]\right|^m . \qquad (12.2)$$

In Eq. 12.2, the following points are important:

1. stress exponent m characterizes the dependence of the inelastic deformation on the stress level in Network (B);
2. the stretch exponent c characterizes the dependence of the rate of inelastic deformation on the current magnitude of inelastic deformation and must be between 0 and negative 1 ($-1 < c < 0$);
3. the viscosity scaling constant Z defines the absolute magnitude of the inelastic deformation;
4. the elastic stretch component $\dot{\lambda}_B^e$ is found by numerically solving the Eq. 12.2;
5. δ ($\delta < 0.001$) is a small positive number added to avoid singularities in the inelastic stretch rate when the inelastic stretch is close to unity.
6. the axial principal stretch can be assumed to be related to engineering strain, $\lambda = 1 + \varepsilon$ (for uniaxial loading of a tissue assumed to be anisotropic).

Using Eqs. 12.1 and 12.2 above with the six parameters (μ_A, μ_B, Z, m, c, α), the two-network model predicts the total nominal stress σ for any input stretch λ_u and stretch rate $\dot{\lambda}_u$ using the equation:

$$\sigma = \frac{2\mu_A}{\alpha}\left[\lambda_u^{\alpha-1} - \lambda_u^{-\frac{\alpha}{2}-1}\right] + \frac{2\mu_B}{\alpha}\left[(\lambda_B^e)^{\alpha-1} - (\lambda_B^e)^{-\frac{\alpha}{2}-1}\right] \quad (12.3)$$

The development of the three-network model results in similar expressions with the additional terms for Network C. This includes a duplicate of Eq. 12.2 but for Network C, replacing 'B' for 'C' for $\dot{\lambda}_C^e$. The resulting parameters for the three-network model are: $\mu_{A,}$, μ_B, μ_C, Z_B, Z_C, m_B, m_C, c_B, c_C, α.

2. IMPLEMENTATION USING MATLAB AND FORTRAN

The two- and three-network models above, as presented in the cited literature, were implemented in MATLAB and the elastic stretch rate for both Network B and Network C were solved numerically using the built-in MATLAB function `ode15s`. This built-in solver is versatile, as well as being useful in solving very stiff systems.

However, using this solver makes interacting with the model difficult. For example, if part of a larger model, muscles may change stretch at any time in no pre-known fashion. Further, the solver keeps the model dependent solely on MATLAB. To use it outside of MATLAB, a more generic numerical solver had to be used. Thus, a generic fourth-order Runge-Kutta routine (`RKfor`) was ported into MATLAB from a current rendition in FORTRAN (see also APDL version[12]).

The `RKfor` routine has a set time step in the solution. While adaptive time steps can save computation time, a set time step is required for time synchronization for future imbedding into a larger speech simulator. For the two-network model, a sampling rate of 1/250 per second was adequate for solutions. The three-network model required a higher sampling rate (smaller solution time step) of approximately 1/1000 second to keep up with simulating the physics.

2.1 Two-Network Model

The two-network Ogden model was implemented in MATLAB into a single script with all supporting functions built in. The script (`ControlMuscleGuessOgden2net.m`) can be downloaded with this technical memo. To run the script, type its name in the MATLAB command window. The result is a figure window with a simulated stress σ as a function of time and simulated stress-strain curve (Figure 12.2 below).

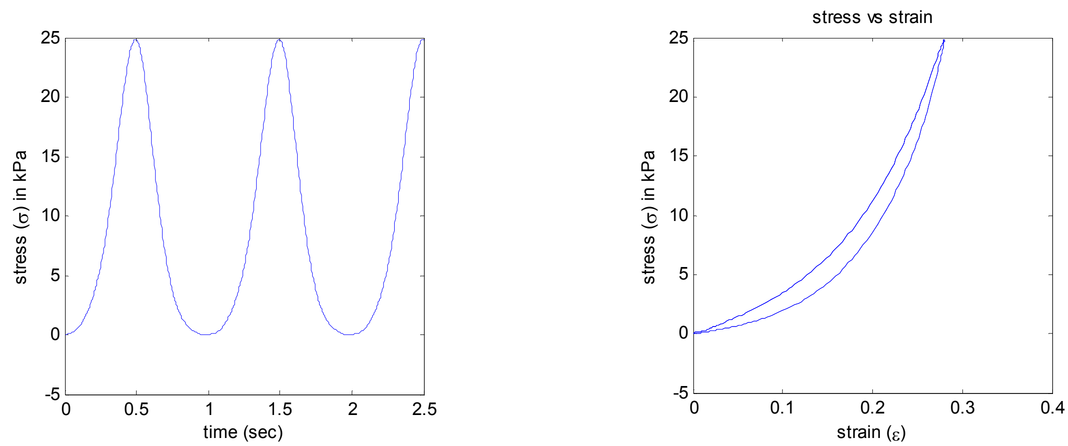


Figure 12.2. Simulated stress for a cyclic strain for the two-network model. (Left) stress as a function of time. (Right) stress-strain curve.

The six parameters of the Ogden model can be varied in lines 12-18. The input stretch λ_u and stretch rate $\dot{\lambda}_u$ can be found in lines 27-30. For this case, a cyclic strain of 30% was used, but any strain could be used. The script is self-contained with all the dependent functions included. The script contains two dependent functions, `model_bananafunc` (where the equations above are implemented) and `RKfor` (a fourth-order Runge-Kutta routine). The function `model_bananafunc` solves the differential equation for elastic stretch rate $\dot{\lambda}_B^e$ of Network B given by Eq. (12.2) numerically using the function `RKfor` to obtain the elastic stretch λ_B^e of Network B. It then computes the total nominal stress σ as given by Eq. 12.3 below.

The two-network model was ported to FORTRAN. Its implementation is very similar to the MATLAB version. The `MuscleGuess.F` is the main program that calls the function `model_bananafunction`, which computes the stress using the Ogden model for a given stretch and its derivative. The function calls another function `RKfor.F90` to solve the Ogden model numerically. These functions are all available for download with this technical note.

2.2 Three-Network Model

The three-network Ogden model was implemented in MATLAB into a single script with all supporting functions built in. The script (`ControlMuscleGuessOgden3net.m`) can be downloaded with this technical memo. To run the script, type the name of the script in the MATLAB command window. While it is similar to the two-network model in that the result is a figure window with a simulated stress σ as a function of time, the contribution of the third network adds a long-term creep (relaxation) to the stress (Figure 12.3).

Although the three-network model is yet to be ported to FORTRAN, it will be in the future.

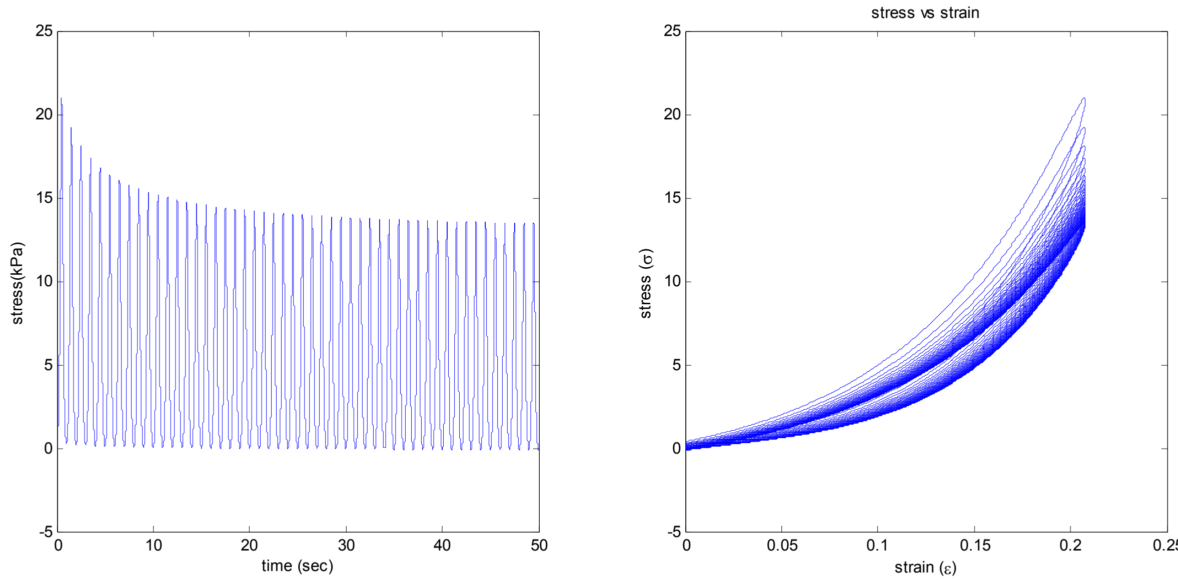


Figure 12.3. Simulated stress for a cyclic strain for the three-network model. (Left) stress as a function of time. (Right) stress-strain curve.

3. OPTIMIZATION AND SENSITIVITIES

The Ogden type models' sensitivity to experimental error and to previous models of axial properties[6] were optimized using an automatic parameter optimization strategy to search for parameters matching actual data.

3.1 Optimization of Parameters

Using the same technique for the optimization of parameters as discussed elsewhere (Hunter & Titze, 2007), the two-network model parameters were optimized to human lamina propria stress strain data. At the MATLAB command window, executing `ControlSimpleFitOgden.m` starts the process. For this example, the 10th and 11th cycle of the data in the file `left_cover_4v_66-130cycle.xls` are loaded, and then the Ogden model parameters are adjusted until a good fit exists between the actual stress and the predicted stress.

The output is starting guess values for the parameters and the end optimized values. Also given on completion are *goodness-of-fit* metrics. The program uses Nelder and Mead Simplex method (NMSM), a direct search method to optimize the parameters (Hunter & Titze, 2007). The starting guess values for the six parameters of the Ogden model can be varied in lines 49-54. The first number is the starting parameter value and second and third numbers are the lower and upper range to search respectively. The scripts are available for download. For the example given above, the result of the optimization would be:

```
Elapse time: 135.086 sec.
Elapse time: 135.123 sec.
 -------------------------------------------------------------
#fun      ---    965
#itt      ---    624
Var         Start      Fit
```

Z	0.0011	0.00095442
c	-0.99792	-1
m	1.2118	1.4098
mu_A	29.165	26.7658
mu_B	193.6132	322.5482
a	19.6836	a20.1869

Figure 12.4 below is an example of such an optimization.

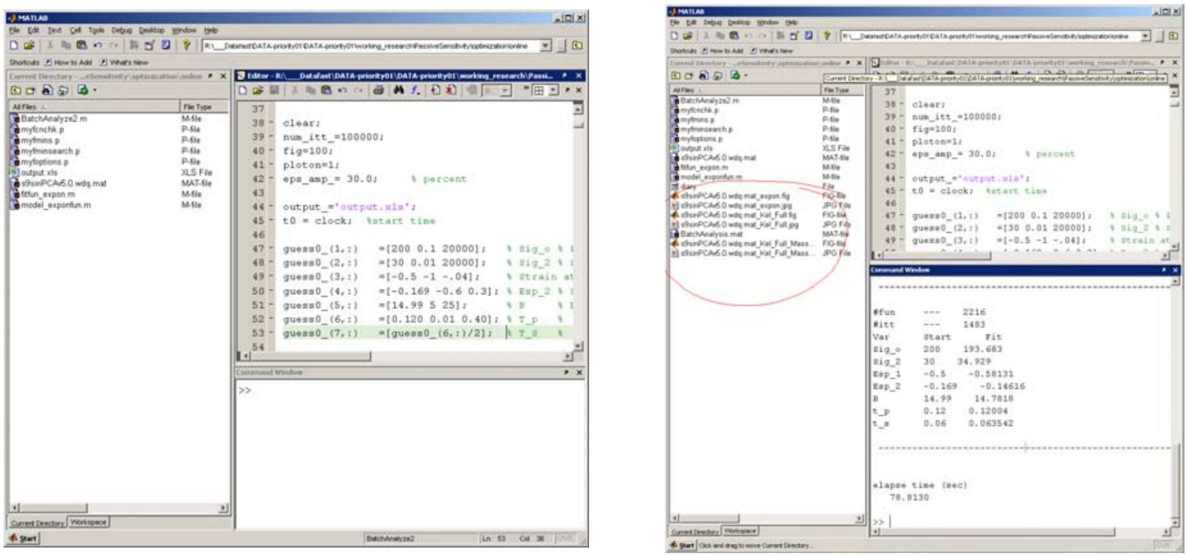


Figure 12.4. LEFT: pre-optimization seed values and maximal number of iterations. RIGHT: the result of the batch optimization. The graphical results of each optimization is saved to graphic files, circled in the left pain of right figure.

3.2 Sensitivity Analysis

Because of the laryngeal tissues' small size, their passive tissue properties are particularly susceptible to small errors and uncertainties in laboratory measurements. In a previous study (Perlman, Titze, & Cooper, 1984), it was demonstrated that the most significant of these are uncertainties/errors in length and cross-sectional area measurements. The length uncertainties/errors were shown to be about ±1/3 mm, potentially resulting in an 8% error in Young's Modulus calculation.

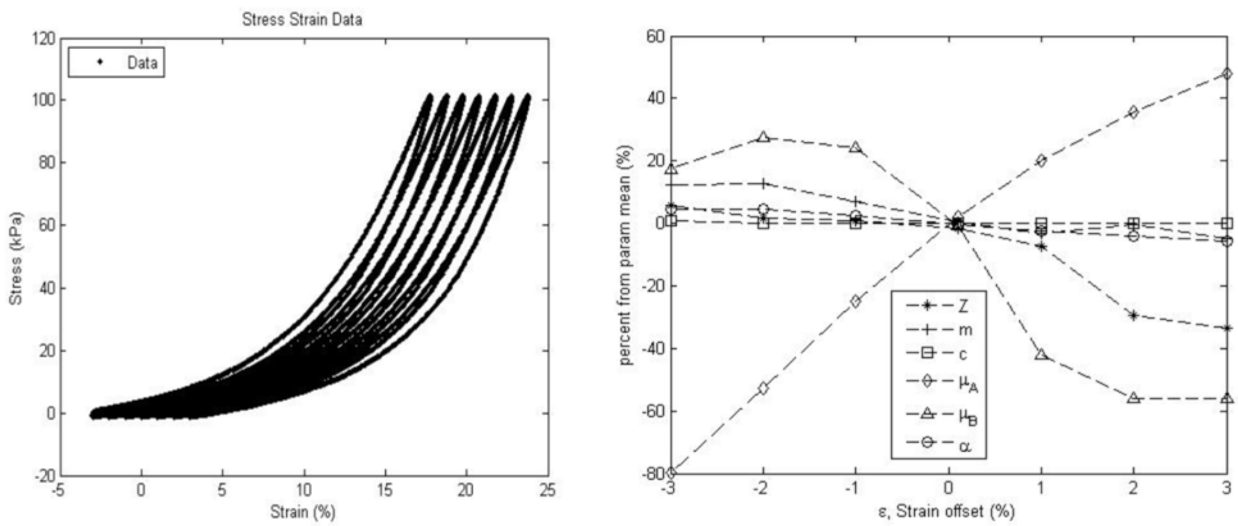
Using this uncertainty as a basis for investigation, the same data used above was modified by applying a strain offset. The model parameters were then optimized to that adjusted data. The offset was ±3%. Figure 12.5 below illustrates the original stress/strain data and the perturbed cases, which simulate the potential uncertainties/errors.

The metric shown in Figure 12.5.b below is the percent of the non-offset parameter optimization as shown in Eq 12.4.

$$percent\ from\ parameter\ mean\ (\%) = \frac{\xi(no\ offset) - \xi(offset)}{\xi(no\ offset)} \times 100 \qquad (12.4)$$

Optimized parameters allowed the model to match the perturbed cases with some of the parameters being insensitive to the perturbations (c, α) and others being sensitive to perturbations ($\mu_{A,}$, μ_B, Z, m) in the following ways:

1. Z is not sensitive to negative perturbations but is sensitive to positive perturbations
2. m is sensitive to negative perturbations but is not sensitive to positive perturbations
3. μ_A has almost linear sensitivity to strain perturbation
4. μ_B was affected by +30% at perturbations below zero and -50% at perturbations above zero.



(a) (b)

Figure 12.5. Strain offset, Condition 1; (a) Strain offset, (b) parameter dependence on strain offset

Conclusions

This technical memo discussed the two- and three-network Ogden models as presented in the literature to capture the characteristics of vocal fold tissue under changing strain environments. The two models were implemented in MATLAB in such a way that their solutions were not dependent on internal MATLAB functions so that they could be ported to other analysis languages.

Sensitivity analysis of the two-network model showed that two parameters were robust to potential errors in strain with four parameters highly sensitive to errors in strain. This sensitivity was dependent on the current optimization routines. It was found that the current optimization implementation `ControlSimpleFitOgden.m` is able to fit the data well only if the stress values are less than 100 kPa for a strain of 30%. If the stress values are beyond this level at a strain of 30%, the Ogden model does a poor job fitting the data using the current optimization protocol. Whether this is because of the parameter search method or characteristics of the model itself is currently being investigated.

Optimization as it is implemented (i.e., when strain in a data set is above 30%) is highly dependent on initial guess and boundary conditions (below 30% and robust optimization). The initial values of the parameters have to be chosen so that (1) the curve looks similar in shape as the given data; and (2) the stretch rate does not become singular. Further, the boundary values must be chosen so that the optimized parameters are well within the chosen range. If the optimized values reach the boundary values, the sensitivity analysis program does not perform well as the chance of stretch rate becoming singular in successive iterations is high.

The above issue may be partially dependent on the method by which the Ogden model has been implemented because it is prone to singularities in the inelastic stretch rate or when the inelastic stretch is close to one. The initial value of inelastic stretch, which is given in line 71 of `ControlMuscleGuessOgden2net.m` script, has to be adjusted if the inelastic stretch rate becomes singular or the initial parameter values have to be adjusted to avoid singularities. Each of these issues need to be addressed before imbedding the Ogden models into larger speech simulation models.

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